



Analytical Carbon Cycle Impulse Response Function

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Abstract

The purpose of this paper is to derive an analytical impulse response function (IRF), for the carbon cycle between atmosphere and sea. The analysis is starting from the Box-Diffusion model (BDM) given by Oeschger et al. The BDM is also the underlying model for the “sum of exponentials – IRF” presently used in many papers to describe the atmosphere/sea carbon cycle.

10 to 100 years is a relevant time scale when discussing the effect of anthropogenic emissions on the atmospheric carbon dioxide. We show that on this time scale there is an analytical IRF. Our view is that this analytical IRF is a better representation of the physics in the system than the presently used “sum of exponentials” IRF.

Another important conclusion is that there is only one characteristic time constant. All physics in the BDM can thus be represented with one characteristic time and an analytical IRF. With the values used in the BDM the characteristic time will be in the interval 80 – 145 years. This value is strongly dependent on assumed values of Revelle factor and eddy diffusion constant.

Keywords: Carbon Cycle Models; Bern model; Impulse response function; Box models; Diffusion;

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1. Introduction

Carbon cycle models are important tools for understanding the atmospheric concentrations of carbon dioxide.

A central model in this context is the so-called Bern Model. It has gradually been developed over the last 50 years. One can trace the basic ideas behind the model to a paper by Suess and Revelle (1957) [1].

The present foundation for the Bern model is a work by Oeschger et al. (1975) [2]. The model was further developed by Siegenthaler (1983) [3], who included a formalism for direct contact between the atmosphere and the so-called deep sea.

Maier-Reimer et al. (1987) [4] introduce the concept of impulse response function (IRF), “Transport and storage of CO₂ in the ocean”. They fitted a five-term exponential sum to the numerically calculated impulse response for the Box Diffusion model.

This five-term exponential impulse response function is commonly referred to in modern articles about Carbon Cycle, e.g Joos et al. (2013) [5]. There have been later modifications to this IRF, but the basic characteristics of Maier-Reimer et al’s IRF are representative for presently used

models.

A very short summary of the evolution of today's carbon cycle models is:

- Suess and Revell, they introduce the so called Revelle factor
- Oeschger et al., developed the Box Diffusion Model
- Maier-Reimer et al., introduced the impulse response function (IRF)

This work will describe a mathematical analysis of the base model – Oeschger et al.'s (1975) [2] Box Diffusion Model which is the foundation for today's Carbon Cycle models.

The aim with this paper is to present an analytically derived IRF, instead of the curve fitted IRF introduced by Maier-Reimer et al. (1987) [4].

We will also show that the IRF of the Box Diffusion Model (BDM) is strongly dependent on the Revelle factor. The Revelle factor describes the solubility of $CO_2(g)$ as function of total dissolved inorganic carbon (DIC). The Revelle factor was first calculated by Bolin and Eriksson (1959) [8]. They reached the conclusion that for a system with sea water and carbon dioxide containing atmosphere the R-value is 12.5. Oeschger et al. use an R-value of 10. Bolin and Eriksson also conclude that the R-value for a system with sea water, limestone (in the sea water), and carbon dioxide containing atmosphere is 2.4.

An R-value equal to 1 means that the carbon outflux from the sea is proportional to the carbon content in the sea.

2. The “Box Diffusion Model”

The BDM (Box Diffusion Model) consists of two boxes, atmosphere, and surface sea (also called mixed layer), the deep sea and the biosphere. The concentration of carbon dioxide in a box is uniform within the box.

Carbon dioxide in the mixed layer is transported to the deep sea by “eddy diffusion”, and the transport within the deep sea is also controlled by so called eddy diffusion. Eddy diffusion is modelled by the diffusion equation, with a constant diffusion coefficient.

In this analysis we will omit the biosphere, which is equivalent to set the “biological growth factor” to zero in the BDM.

The reason why we omit the biosphere is that our analytical IRF is compared with Maier-Reimer's (1987) [4] curve fitted exponential IRF. That IRF is only for the atmosphere-ocean system, why the biosphere is not needed for comparisons of the mathematics.

If one wants a complete carbon cycle model that should of course include the biosphere. Future work may develop models with our analytical IRF (representing the interaction between atmosphere and sea) and the biosphere included.

The BDM uses the following governing equations.

$$\frac{dn_a}{dt} = p + k_{ma}(N_m + R * n_m) - k_{am}(N_a + n_a) \quad (1)$$

$$\frac{dn_m}{dt} = k_{am}(N_a + n_a) - k_{ma}(N_m + R * n_m) + K * \frac{\partial c_d}{\partial z} (z = 0) \quad (2)$$

$$\frac{\partial c_d}{\partial t} = K \frac{\partial^2 c_d}{dz^2} \quad (3)$$

$$\frac{N_m + n_m}{h_m} = C_d + c_d \text{ för } z = 0 \quad (4)$$

Eq. (4) follows from the boundary condition; carbon concentration at the top of deep sea ($z=0$) equals the concentration in the mixed layer

$$K \frac{\partial c_d}{dz} = 0 \text{ för } z = h_d; \text{ no flux through deep sea floor} \quad (5)$$

The symbols in the equations are:

$$N_i + n_i(t) = \frac{\text{total amount of } CO_2 \text{ in reservoare } i}{\text{surface area}}, i$$

= a (atmosphere), m (mixed layer), d (deep sea),

N is the preindustrial equilibrium value (constant) and n is the deviation [$\frac{mol}{m^2}$]

$C_d + c_d(t)$

= CO_2 concentration in deep sea, C_d is constant and depth independent [$\frac{mol}{m^3}$]

$$p(t) = \frac{(CO_2 \text{ production rate})}{\text{surface area}} \left[\frac{mol}{m^2 y} \right]$$

h_m = depth of mixed layer = 75 m

h_d = depth of deep sea = 3725 m

$$k_{am} = \text{exchange rate atmosphere} - \text{mixed layer} = \frac{1}{7.53 \text{ yr}}$$

$$k_{ma} = \text{exchange rate mixed layer} - \text{atmosphere} = \frac{1}{9.53 \text{ yr}}$$

$$K = \text{eddy diffusion constant} \approx 4000 \frac{m^2}{yr}$$

R = Revell factor ≈ 10

$$k_{am} * N_a = k_{ma} * N_m \quad (6)$$

The notation is the same as in the work of Oeschger et al. (1975) [2].

3. Analysis of the timescales in the Box Diffusion Model

The above are the basic equations of the Bern model. We now turn to our analysis of these equations. The following are thus our calculations, and not described by Oeschger et al. (1975) [2].

Consider only the atmospheric – mixed layer system (i.e., no diffusion to the deep sea). Use Eq. (6) in Eq. (1) and (2).

$$\frac{dn_a}{dt} = p + k_{ma} * R * n_m - k_{am} * n_a \quad (7)$$

$$\frac{dn_m}{dt} = k_{am} * n_a - k_{ma} * R * n_m \quad (8)$$

For p = a pulse with amplitude (area) 1, this equation system has the solution:

$$n_a = \frac{k_{ma} * R}{k_{am} + k_{ma} * R} * \left(1 + \frac{k_{am}}{k_{ma} * R} * e^{-(k_{am} + k_{ma} * R) * t} \right) \quad (9)$$

$$n_m = \frac{k_{am}}{k_{am} + k_{ma} * R} * \left(1 - e^{-(k_{am} + k_{ma} * R) * t} \right) \quad (10)$$

With current values of the constants, the time constant becomes about 1 year. The dynamic of this system is fast, i.e., the equilibrium sets in such a short time that one can consider the atmosphere-mixed layer as equilibrium for the times of interest, i.e., tens of years.

It is a reasonable approximation to set these to their equilibrium levels, i.e., the amount present in the total reservoir atmosphere + mixed layer is distributed according to:

$$n_a = \frac{k_{ma} * R}{k_{am} + k_{ma} * R} \approx 0.89 \quad (11)$$

This means that about 89 % of the total content in the combined reservoir is in the atmosphere.

$$n_m = \frac{k_{am}}{k_{am} + k_{ma} * R} \approx 0.11 \quad (12)$$

About 11 % of the total content in the combined reservoir is in the mixed layer.

The relation between mixed layer and atmosphere is given by:

$$n_m = \frac{k_{am}}{(k_{ma} * R)} * n_a = \alpha * n_a \quad (13)$$

Where $\alpha \approx 0.13$.

Atmosphere – mixed layer now becomes like one reservoir emptied by diffusion from the mixed layer down to the deep sea.

Under the approximation of instant equilibrium between atmosphere and mixed layer we get the following basic equations.

$$\frac{d(n_a + n_m)}{dt} = p + K * \frac{\partial c_d}{\partial z}(z = 0) \quad (14)$$

Use Eq. (13) in Eq. (14), which gives:

$$\frac{dn_a}{dt} = \frac{k_{ma} * R}{k_{am} + k_{ma} * R} * \left(p + K * \frac{\partial c_d}{\partial z}(z = 0) \right) = \beta * \left(p + K * \frac{\partial c_d}{\partial z}(z = 0) \right) \quad (15)$$

$$\beta = \frac{k_{ma} * R}{k_{am} + k_{ma} * R} \approx 0.89 \quad (16)$$

$$\frac{\partial c_d}{\partial t} = K * \frac{\partial^2 c_d}{\partial z^2} \quad (17)$$

With the boundary conditions:

$$c_d = \frac{n_m}{h_m}, \quad z = 0 \quad (18)$$

The carbon concentration in the deep sea is equal to the concentration in the mixed layer at the boundary.

$$\frac{\partial c_d}{\partial z} = 0, \quad z = h_d \quad (19)$$

The implication is that there is no net flow through the seabed.

Since diffusion is relatively slow, i.e., it takes a while for the concentration at the bottom of the deep sea to change, the second boundary condition, Eq. (19), can be ignored when one wants to

get an idea of the system's behavior over a 100-year period. It will be the same as viewing the deep sea as infinite.

Note that the system is now a kind of "1-box model" that is emptied by diffusion. The Bern model is thus a diffusion model.

4. Derivation of an analytical Impulse Response Function

Eq. (15) can be solved analytically, under the assumption that the diffusion time in the deep sea is long compared to the time scale of interest.

Now we apply Laplace transformation of the equations. Denote the Laplace transform by the corresponding capital letter and the transform variable with s . For example, $N_a(s)$ is thus the Laplace transform of $n_a(t)$ given by equation 20, and is not related to the equilibrium value N_a .

$$N_a(s) = \int_0^{\infty} e^{-st} * n_a(t) dt \quad (20)$$

Eq. (17) becomes:

$$s * C_d(s) = K * \frac{\partial^2 C_d(s)}{dz^2} \quad (21)$$

Which has the general solution:

$$C_d(s) = a1(s) * e^{-z * \sqrt{\frac{s}{K}}} + a2(s) * e^{z * \sqrt{\frac{s}{K}}} \quad (22)$$

Since the concentration must go towards zero as the depth moves towards infinity, it follows that $a2(s)=0$.

Eq. (18) gives $a1(s) = N_m(s)/h_m$. Insert $C_d(s)$ as given by Eq. (22) into the Laplace transform of Eq. (15),

$$s * N_a(s) = \beta * \left(P(s) - \frac{N_m(s)}{h_m} * \sqrt{s * K} \right) = \beta * \left(P(s) - N_a(s) * \frac{\alpha}{h_m} * \sqrt{s * K} \right) \quad (23)$$

Let

$$\gamma = \beta * \alpha * \sqrt{K}/h_m \propto \sqrt{K}/R \quad (24)$$

We now get the equation,

$$N_a(s) = \beta * P(s)/(s + \gamma * \sqrt{s}) \quad (25)$$

Consider the case that the carbon dioxide is added as an impulse with area 1. This means that $P(s) = 1$.

By rewriting the expression:

$$N_a(s) = \frac{\beta * 1}{\gamma} * \left(\frac{1}{\sqrt{s}} - \frac{1}{\sqrt{s} + \gamma} \right) \quad (26)$$

Now, define a time constant $\varphi = 1/\gamma^2$, or

$$\varphi = \left(\frac{h_m * (k_{am} + k_{ma} * R)}{k_{am} * \sqrt{K}} \right)^2 \quad (27)$$

The expression can now be inverse transformed using tables. The solution is:

$$n_a(t) = \beta * e^{t/\varphi} * erfc(\sqrt{t/\varphi}) \quad (28)$$

This is thus the impulse response function of the Bern model, under the assumption of an infinite deep sea and an instant equilibrium between atmosphere and mixed layer. It represents the dynamics of the BDM on a time scale ten to hundreds of years.

It is this expression that in later articles is approximated with a sum of exponential functions.

Using the values given in Oeschger et al's (1975) [2] paper, φ will have a value in the range of 80 – 145 years, with an expected value of 111 years. It should however be noted that Oeschger et al. assume that the Revelle factor has a value about 10 with high certainty. A lower value of the Revelle factor will have a significant effect on the φ – value.

Limit values can be checked:

When $t=0$, $n_a = \beta$ which is the pulse distribution between atmosphere and mixed layer.

When t becomes large, the approximate expression of complementary error function can be used,

$$erfc(\gamma\sqrt{t}) \approx \frac{e^{-\gamma^2 t}}{\gamma\sqrt{\pi t}}$$

From this one can see that the carbon dioxide content of the atmosphere will go towards zero as $1/\sqrt{time}$ (assuming infinite deep sea!) which is consistent with a diffusion process.

4.1 How long does the "infinite deep-sea approximation" apply?

A rough estimate is to take the "diffusion distance" $2 * \sqrt{K * t}$ and compare with the sea depth of 3725 m. It takes about 1800 years for the "diffusion distance" to be comparable to the depth of the sea. When looking at a period of about 100 years, "infinite deep sea" is a good description of the dynamics of the system.

4.2 Equilibrium

If one considers that the sea is not infinite, a balance will of course occur where the residual value of the atmosphere's carbon dioxide content is greater than zero.

This equilibrium can be easily calculated:

From the boundary condition that the carbon dioxide flow at the bottom of the sea should be zero, it follows that the concentration c_d must be a constant at equilibrium.

The equilibrium atmospheric content can be calculated by inserting Eq. (13) and Eq. (18) into Eq. (29) below (the sum of the reservoir's content must equal the injected pulse).

$$c_d * h_d + n_m + n_a = 1 \quad (29)$$

After a pulse injection (of area 1) in the atmosphere,

$$n_{a_{eq}} = \frac{1}{1 + \alpha * \left(1 + \frac{h_d}{h_m}\right)} \approx 0.13 \quad (30)$$

The perpetual contribution to the atmosphere is reinforced by the Revelle factor. Instead of about 2 % remaining (current equilibrium distribution atmosphere – ocean), about 13 % will stay forever.

At a first glance at Eq. (30), it looks as if the equilibrium atmospheric content is dependent on the depth of the mixed layer. This is however not really the case.

$$k_{ma} = 1/\text{residence time in the mixed layer} = 1/\tau_m$$

It is reasonable to assume that the residence time is proportional to volume, meaning:

$$\tau_m = \mu * h_m, \text{ where } \mu \text{ is a constant}$$

Inserting this into Eq. (30) gives:

$$n_{aeq} = \frac{1}{1 + \alpha * \left(1 + \frac{h_d}{h_m}\right)} = \frac{R}{R + k_{am} * \mu * (h_m + h_d)} \quad (31)$$

This equation is independent of where one chooses to end mixed layer and start deep sea.

4.3 The biosphere

The biosphere is included in the BDM. The full equation for the atmosphere in Oeschger et al.'s work (1975) [2] is:

$$\frac{dn_a}{dt} = p + k_{ma}(N_m + R * n_m) - k_{am}(N_a + n_a) - \varepsilon * F * \left(\frac{n_a(t)}{N_a} - \frac{n_a(t - \tau)}{N_a}\right) \quad (32)$$

where

ε = biological growth factor

F = flux to biosphere [moles/year]

τ = biospheric time delay for CO₂

In the derivation of the analytical IRF we set $\varepsilon = 0$, which is Eq (1) and is equivalent to a constant biosphere.

The purpose with this paper is to derive an analytical IRF for the system atmosphere – sea. This analytical IRF is comparable to the curve fitted exponential IRF derived by Maier-Reimer (1987) [4], which is frequently used in present carbon cycle models, e.g., Joos et al. (2013) [5]. The curve fitted IRF is calculated from numerical simulations with $\varepsilon = 0$. This means that the curve fitted IRF and the analytical IRF are based on the same governing equations.

The biosphere (as modelled in Oeschger et al.'s work) can easily be included in the analytical IRF if we do either of the following approximations:

1- assume that $n_a(t) - n_a(t - \tau) \approx \text{constant} * \tau$ (atmospheric increase is linear)

2- assume that $(n_a(t) - n_a(t - \tau))/\tau \approx \frac{dn_a}{dt}(t)$

5. The development of the exponential IRF

Maier-Reimer et al. (1987) [4] introduced the Impulse Response Function (IRF) concept for carbon cycle models.

They fitted a sum of exponential functions to a numerical calculated impulse response of the Bern model. The idea was to get an expression that could be used in analytical analysis.

They obtain the following expression for the IRF (for small disturbances):

$$IRF = 0.131 + 0.201 * e^{-\frac{t}{363}} + 0.321 * e^{-\frac{t}{74}} + 0.249 * e^{-\frac{t}{17.3}} + 0.098 * e^{-\frac{t}{1.9}} \quad (33)$$

Following their article, they comment that this expression is in good agreement with numerically calculated IRF based on Siegenthaler's work (1983) [3].

Siegenthaler (1983) [3] use the same basic equations as Oeschger et al.(1975) [2], but includes a direct contact between the deep see and the atmosphere in the model.

The above IRF is compared to Siegenthaler's reference case with zero direct contact between deep sea and atmosphere, which is equivalent to Oeschger's model (Bern model).

It can thus be concluded that the IRF derived by Maier-Reimer et al. (1987) [4] is based on the governing equations developed in Oeschger et al.'s work.

Figure 1 shows the impulse response based on Eq. (28) compared to the IRF given by Eq. (33). The agreement is good.

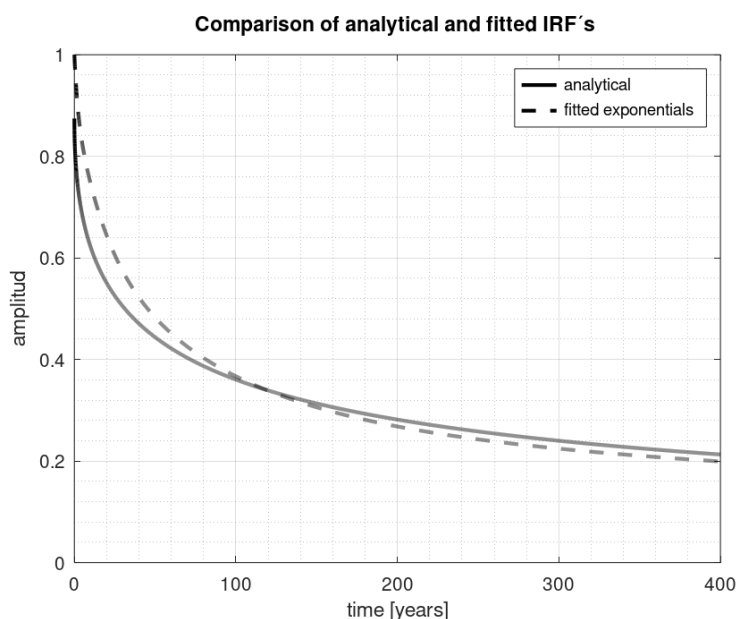


Figure 1 – Impulse response according to equation 28 compared to IRF derived by Maier-Reimer et. al {4}.

Figure 2 shows how the analytical impulse response given by Eq. (28) depends on the Revelle factor.

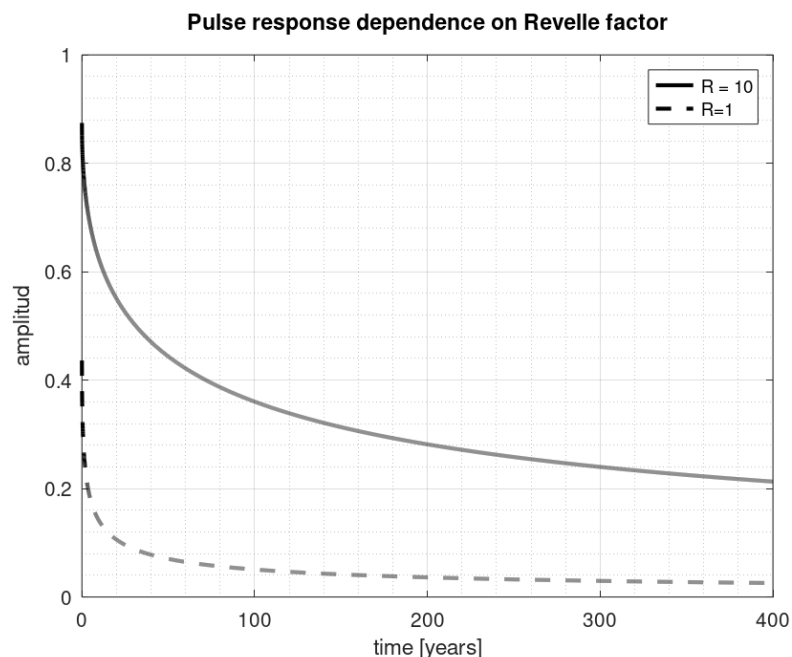


Figure 2 – Impulse response according to Eq. (28), with different Revelle values.

The IRF derived by Reimer-Maier is very similar to IRF's used in later articles/ models. E.g., ACC2, Tanaka et al. (2007) [6], use an IRF of the form:

$$IRF = 0.132 + 0.311 * e^{-\frac{t}{236.5}} + 0.253 * e^{-\frac{t}{59.5}} + 0.209 * e^{-\frac{t}{12.2}} + 0.095 * e^{-\frac{t}{1.27}} \quad (34)$$

There is no direct exchange between deep sea and the atmosphere in the ACC2 model, which means that it is consistent with Oeschger et al.'s Box-Diffusion model.

The wording in the ACC2 paper is “ACC2 accounts for the influence of the solubility pump from the temperature change by calculating the thermodynamic equilibria explicitly, but it cannot resolve the influence from the thermohaline circulation and the ocean ventilation (Falkowski et al., 2000)”

It is clear that presently used IRF's are based on Oeschger et al.'s work, and that there is no direct contact between atmosphere and deep sea included in the IRF's.

6. Anthropogenic contribution to atmospheric carbon dioxide level

The analytical IRF derived in the previous section, Eq. (28), can be used to calculate the anthropogenic contribution to the increased carbon dioxide content in the atmosphere.

The accumulated atmospheric carbon dioxide is given by the convolution.

$$C_{atm}(t) = C_{atm0} + \int_{t_0}^t IRF(t - \tau) * H(\tau) d\tau \quad (35)$$

C_{atm} = atmospheric CO₂ content in ppm

C_{atm0} = 280 ppm = reference level at year 1750

$H(t)$ = anthropogenic contribution

According to Harde (2019) [7] the anthropogenic contribution can be described by the following expression.

$$H(t) = h_0 * \left(e^{\frac{t-t_0}{\delta}} + h_1 \right) \quad (36)$$

Where:

$h_0 = 0.026 \text{ ppm/yr}$

$\delta = 50 \text{ yr}$

$t_0 = 1750$

$h_1 = 4$

Inserting the analytical IRF given by Eq. (28), and Eq. (36) into Eq. (35) makes it possible to calculate how much human emissions has added to the atmospheric concentration as function of the Revelle factor.

Eq. (35) must be numerically integrated. The value is 160 ppm, using the data in the BDM. This is higher than the observed atmospheric increase. The reason can be that the BDM underestimates sea absorption, or that there are other sinks than the ocean. E.g., the biosphere. If one assumes that the emission from land/biosphere has increased due to increased temperature, then it is likely that the BDM underestimates the sea absorption.

Eq. (35) is strongly dependent on the Revelle factor (R-value).

Since it is possible to assume different R-values, Eq. (35) is analyzed with respect to this parameter.

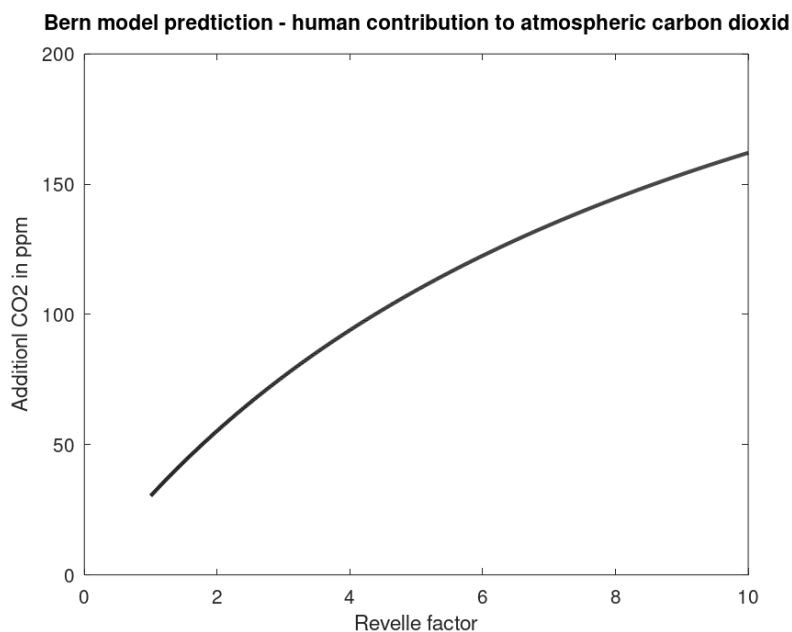


Figure 3. Atmospheric carbon dioxide increases due to human emissions, from 1750 – 2020, as function of the Revelle factor.

Figure 3 shows the increase in atmospheric carbon dioxide concentration (due to anthropogenic emissions) year 2020 as function of the Revelle factor (R-value).

The calculation is based on:

- Oeschger et al.'s Box-Diffusion model (Bern model)
- Harde's equation for human emissions (36)

7. Summary and Discussion

We have developed an analytical expression for the Bern model impulse response function (IRF). The derivation is based on two assumptions:

- neglect the time to establish equilibrium between the atmosphere and mixed layer. The argument is that the timescale for equilibrium atmosphere – mixed layer is short (about 1 year) compared to the timescale of interest.
- the deep sea has infinite depth. This approximation should be reasonable for times up to several hundreds of years.

The derived IRF is consistent for a system dominated by diffusion.

The analytical IRF also has a physical meaning. Our view is that this is an advantage since it is difficult to give a physical interpretation of a curve fitted equation.

The present use of the curve fitted exponential IRF is somewhat misleading. Since this type of IRF is correct for a box model, it is easy to view the underlying model as a box model. It is not. It is a diffusion model. Oeschger et al. comment that box models do not agree with observations,

why they propose a diffusion model instead of a box model. When one fits a box model IRF to the BDM it lacks physical meaning.

The exponential IRF gives the impression that there are some outflows which are very slow (exponentials with long time constant) and that a certain fraction of the outflow must pass through those “slow” outflows. That is of course not the case.

The analytical IRF derived in this paper reflects what actually is going on in the BDM. There is a fast but limited uptake in the surface sea (restricted by the Revelle factor). The additional CO_2 in the surface sea is transported away by a rather slow diffusion process. Since only a small fraction of the excess CO_2 is in the surface sea – only a small part of the addition is exposed to the downward diffusion.

One important feature is that the derived IRF only has one characteristic time constant, Eq. (27). That makes it possible to easily evaluate the effect of the parameters on the system dynamics.

It is a fact that the biosphere has a significant impact on the atmospheres CO_2 level, and the assumption about $\varepsilon = 0$ is a simplification. The same assumption is, however, done in Maier-Reimer et al.'s work when they introduce the exponential IRF.

The biosphere can rather easily be included in the present work if one assumes that the biosphere reacts to the change rate of atmospheric CO_2 (the time derivative). In the BDM they assume that the reaction is proportional to the present CO_2 minus the CO_2 some years ago (τ).

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